

# Reliability Based Asset Management Strategy for Concrete Infrastructure

L. Shao<sup>1</sup> and C.Q. Li<sup>2</sup>

<sup>1</sup>Faculty of Civil Engineering and Architecture, Zhejiang University of Technology, P R China

<sup>2</sup>Department of Civil Engineering, University of Greenwich, UK, Email: c.q.li@greenwich.ac.uk

---

## Abstract

To keep civil infrastructure operating at a high level of safety and serviceability, maintenances, such as repairs and strengthening, are necessary. It is also observed that some severely deteriorated concrete structures survive for many years without maintenance. This raises the question of why and how to maintain corrosion affected concrete structures, in particular in the climate of an increasing scarcity of resources. The present paper attempts to formulate an asset management strategy based on risk cost optimization for infrastructure during its whole service life. A time-dependent reliability method is employed to determine the probability of attaining each phase of the service life. To facilitate practical application of the formulated strategy, an algorithm is developed and programmed in a user-friendly manner and followed by a numerical example. It is found in the paper that there exist an optimal number of maintenances for cracking and delamination that returns the minimum total cost for the structure in its whole life. The asset management strategy presented in the paper can help structural engineers, operators and managers make decisions with regard to repairs, strengthening and/or rehabilitation of corrosion affected concrete infrastructure.

*Keywords:* Maintenance; Concrete structures; Reinforcement corrosion, Total cost; Risk, Optimization.

---

## 1. Introduction

Asset management of existing infrastructure has become increasingly important since the aging and deterioration of physical infrastructure will inevitably reduce its load-carrying capacity. This makes the infrastructure increasingly vulnerable over time during its expected service life and poses a potential risk to the public at large. For concrete infrastructure located in saline laden environments, the corrosion of reinforcing steel in concrete is the recognised causal factor for widespread premature and/or unexpected structural failures and significant reduction of service life expectancy [1,2]. Scientifically it is a complicated problem, consisting of a few phases during the service life and involving a large number of interactive contributing factors, such as the environments the infrastructure is exposed; the quality of concrete originally used in its construction, the detailing of structural elements and the loads applied. Socio-economically the consequences of infrastructure failures can be catastrophic, as Hurricane Katrina demonstrated, with human tragedies, environmental disasters and huge economic losses. To keep civil infrastructure operating at a high level of safety and serviceability, maintenances, such as repairs and strengthening, are necessary but maintenance required for corrosion induced deterioration and

damage is generally expensive, disruptive, and involve health and safety risks for infrastructure operators, contractors and the public nearby. It is therefore imperative that a rational asset management strategy be developed to decide when, where and what maintenance is necessary.

On the other hand, it is also observed that some severely deteriorated concrete structures survive for many years without maintenance. This raises the question of why and how to maintain deteriorated infrastructure. With the increasing scarcity of resources, any maintenance for deteriorated infrastructure needs to be evaluated cost-effectively. One solution could be to prolong the overall service life of the deteriorated structures by extending the time period of each phase through intermediate maintenances, provided that the overall safety of the structure is not compromised. This requires a risk cost optimized approach in the development of the asset management strategy.

Various frameworks have been proposed to formulate strategies for inspection, maintenance and decision-making for deteriorated structures, using reliability-based optimization [3,4,5,6,7,8,9]. These strategies have been applied to a variety of structural problems, e.g., to welded connections subjected to cyclic loading in fixed steel offshore structures and to onshore structures such as RC bridges subjected to de-icing salts. It appears that most of these frameworks consider either the ultimate limit state [5,7] or serviceability limit state [4,9,10] individually as the criterion for service life. The whole life behavior over different phases of the service life has not been fully considered in a systematic manner. There exists a gap between “unserviceable” and “unsafe” structures, which may prevent the possible extension of service life. It is in this regard that the present paper attempts to address the issue of asset management strategy considering the whole life behavior of the structure and optimization of risk and cost during the whole life.

The intention of this paper is to formulate an asset management strategy based on the concept of risk cost optimization for infrastructure during its whole service life. Performance-based models are proposed to determine each phase of service life of the corrosion affected infrastructure. A time-dependent reliability method is employed to determine the risk of structural failures in each phase. To facilitate practical application of the formulated strategy, an algorithm is developed and programmed in a user-friendly manner. A numerical example is given to illustrate the application of the proposed asset management strategy to reinforced concrete seawalls. A merit of the proposed asset management strategy is that models used in risk assessment and service life prediction are directly related design criteria used by practitioners.

## 2. Risk Cost Optimization

The underlying principles to develop an asset management strategy for infrastructure are to keep the overall risk of structural failures below an acceptable level throughout its whole service life, whilst intermediate maintenances are carried out to retain its serviceability. To achieve this, it is important to assess structural response in each phase of its service life, which is defined in this paper as a time period, within the whole service life, at the end of which maintenances, e.g., repairs, strengthening or rehabilitation, are required. In the case of corrosion affected concrete infrastructure (Figure 1), the first phase of service life is the time period from the completion of a newly built structure to corrosion initiation in the structure, denoted as  $(0, T_i]$ . Recent research has provided a wealth of evidence that the time to corrosion initiation in practical RC structures (in particular flexural members) in a saline laden environment can be negligibly short in service life consideration e.g. [11,12,13,14,15].

Thus, this phase of service life will not be discussed herein.

The second phase of service life is the time period from the corrosion initiation to corrosion induced concrete cracking, denoted as  $(T_i, T_c]$ . In this paper corrosion induced crack width is used as a

criterion to determine the phase of concrete cracking. The third phase is the time period from concrete cracking to delamination, denoted as  $(T_c, T_d]$ . The fourth phase of service life is the time period from loss of serviceability (cracking or delamination) to loss of strength, denoted as  $(T_c, T_r]$ . In this paper, loss of strength is represented by structural rupture at a critical section of a structural member.

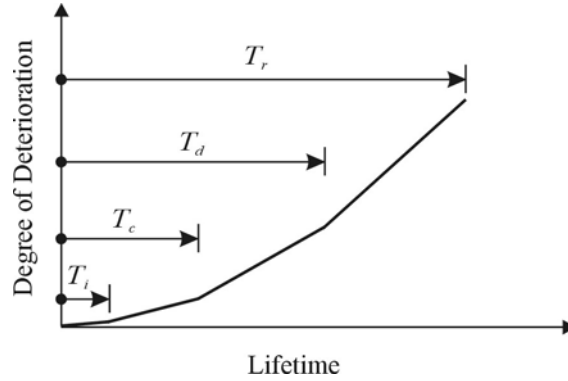


Fig. 1 Phases of service life for corrosion-affected concrete structures

To maintain the safety and serviceability of a structure during its service life, intermediate maintenance for the structure is necessary. Types of maintenance are related to types of structural response. With the model of service life in Figure 1, these include (i) superficial patching for concrete cracking, (ii) major repair for concrete delamination and (iii) overall structural strengthening for rupture (or end of service life). The attainment of each phase is quantified by a probability  $p_c$ ,  $p_d$  or  $p_r$ , respectively. Clearly only when  $p_c$  or  $p_d$  is greater than an acceptable limit respectively will the corresponding maintenance be warranted to achieve cost effectiveness. Similarly,  $p_r$  has to be smaller than an acceptable limit to eliminate undue risk of rupture. With these constraints, the time and number of interventions can be determined through conventional optimization in terms of a total cost,  $C_T$ . Mathematically this can be expressed as:

Minimizing

$$C_T(t_L) = \sum_{i=1}^{n_{mc}} C_c(t_c^i) \cdot p_c(t_c^i) + \sum_{i=1}^{n_{md}} C_d(t_d^i) \cdot p_d(t_d^i) + C_r(t_L) \cdot p_r(t_L) \quad (1)$$

Subject to 
$$p_c(t_c^i) \geq p_{c,a}, p_d(t_d^i) \geq p_{d,a}, p_r(t_L) \leq p_{r,a}$$

where  $C_c$  and  $C_d$  are maintenance costs of concrete cracking and delamination and  $C_r$  is the cost due to structural rupture. All costs are relative to the initial construction cost of the structure so that the data on costs are relatively easy to collect. In Equation (1),  $t_c^i$  and  $t_d^i$  are the time of maintenances for concrete cracking and delamination respectively;  $t_L$  is the time for strengthening or lifetime of the structure;  $n_{mc}$  and  $n_{md}$  are the number of maintenances for concrete cracking and delamination, respectively (corresponding to  $t_c^i$  and  $t_d^i$ ). The design variables in this optimization are  $t_c^i, t_d^i, t_L, n_{mc}$  and  $n_{md}$ . For simplicity, interdependence between cracking and delamination is not included in Equation (1) to achieve feasible practical applications [16]. In Equation (1), the probability terms with subscripts  $a$  are acceptable limits.

The outputs of the optimization will form the basis of asset management strategy for the structure. That is to determine, at minimum total cost and acceptable risk, when ( $t_c^i$  and  $t_d^i$ ) and where ( $n_{mc}$  and  $n_{md}$ ) and what types of maintenance (cracking, delamination or strengthening) are necessary during the service life of the structure and the associated confidence of achieving each phase of the service life ( $p_c$ ,  $p_d$  and  $p_r$ ). It needs to be noted that how to maintain is beyond the scope of the paper assuming that after maintenance the structure is instated to a proportion of its original state.

In Equation (1) the cost terms will include changes in future values. In general, the cost in future values can be determined by [17]

$$C(t) = C(0)(1 + i_r)^t \quad (2)$$

where  $i_r$  is the inflation rate. The risk terms in Equation (1) will be dealt with in the section below and the optimization in the section after.

### 3. Risk Assessment

#### 3.1 Time dependent reliability method.

In assessing the risk of failures for a structure, a performance criterion should be established for the structure. In the theory of structural reliability, this criterion is expressed in the form of a limit state function as follows

$$G(L, S, t) = L(t) - S(t) \quad (3)$$

where  $S(t)$  is the structural response (or load effect),  $L(t)$  is an acceptable limit for structural response (or structural resistance) and  $t$  is time. With the limit state function of Equation (3), the probability of structural failure,  $p_f$ , can be determined by [16]

$$p_f(t) = P[G(L, S, t) \leq 0] = P[S(t) \geq L(t)] \quad (4)$$

where  $P$  denotes the probability of an event.

Equation (4) represents a typical upcrossing problem, which can be solved using the concept of "first passage probability" and expressed as follows [16]

$$p_f(t) = 1 - [1 - p_f(0)]e^{-\int_0^t \nu d\tau} \quad (5)$$

where  $p_f(0)$  is the probability of structural failure at time  $t = 0$  and  $\nu$  is the mean rate for the response process  $S(t)$ , to upcross the threshold,  $L(t)$ . The upcrossing rate in Equation (5) can be determined by the following equation [18] when  $S(t)$  is a Gaussian process and the threshold  $L$  is deterministic,

$$\nu = \nu_{L=\text{det}}^+ = \frac{\sigma_{\dot{S}|S}}{\sigma_S} \phi\left(\frac{L - \mu_S}{\sigma_S}\right) \left\{ \phi\left(\frac{\dot{L} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}}\right) - \frac{\dot{L} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \Phi\left(-\frac{\dot{L} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}}\right) \right\} \quad (6)$$

where  $\nu_{L=\text{det}}^+$  denotes the upcrossing rate when the threshold  $L(t)$  is deterministic,  $\dot{L}$  and  $\dot{S}(t)$  are the time derivative processes of  $S(t)$  and  $L(t)$ ,  $\phi(\cdot)$  and  $\Phi(\cdot)$  are standard normal density and distribution functions respectively,  $\mu$  and  $\sigma$  denote the mean and standard deviation of  $S$  and  $\dot{S}$ , represented by subscripts and " $|$ " denotes the condition. For a given Gaussian stochastic process

with mean function  $\mu_S(t)$  and auto-covariance function  $C_{SS}(t_i, t_j)$  all variables in Equation (6) can be determined, based on the theory of stochastic processes (which will not be repeated here but see, e.g., [16,19]).

To apply Equation (4) to the problem of risk-cost optimization of Equation (1) the main effort lies in developing stochastic models of structural response  $S(t)$ . As formulated in Equation (1), these include (i) corrosion induced concrete cracking; (ii) corrosion induced concrete delamination; and (iii) corrosion induced rupture of a structural member. Each will be dealt with in the next sections.

### 3.2 Corrosion induced concrete cracking.

The practical performance criterion related to concrete cracking is to limit the crack width at the surface of the concrete to an acceptable level. According to Equation (3) this criterion can be expressed as

$$G(w_{cr}, w, t) = w_{cr}(t) - w(t) \quad (7)$$

where  $w(t)$  is the crack width (load effect) at the surface of concrete cover at time  $t$  and  $w_{cr}(t)$  is a critical limit for the crack width. With this limit state function, the probability of structural failure due to concrete cracking can be determined from Equations (5) and (6) with  $w$  replacing  $S$  and  $w_{cr}$  replacing  $L$ . Since it is unlikely that the corrosion induced crack width in concrete exceeds a critical limit at the beginning of structural service, the probability of concrete cracking at  $t = 0$  is zero, i.e.,  $p_c(0) = 0$ . Also, since in most practical applications, the critical limit for crack width  $w_{cr}$  is a constant, prescribed in design codes and standards, Equation (5) can be expressed, after substituting Equation (6), as [16,18]

$$p_c(t) = \int_0^t \frac{\sigma_{w/w}(t)}{\sigma_w(t)} \phi\left(\frac{w_{cr} - \mu_w(t)}{\sigma_w(t)}\right) \left\{ \phi\left(-\frac{\mu_{w/w}(t)}{\sigma_{w/w}(t)}\right) + \frac{\mu_{w/w}(t)}{\sigma_{w/w}(t)} \Phi\left(\frac{\mu_{w/w}(t)}{\sigma_{w/w}(t)}\right) \right\} d\tau \quad (8)$$

In Equation (7), the crack width  $w(t)$  can be modeled as follows

$$w(t) = w_c(t) \cdot \xi_w \quad (9)$$

where  $w_c(t)$  is treated as a pure time function of crack width and  $\xi_w$  is a random variable defined in such a way that its mean is unity, i.e.,  $E(\xi_w) = 1$  and its coefficient of variation is  $\lambda_w$ . [20] developed a formula of corrosion induced crack width  $w_c$  based on the model of thick-wall cylinder (Figure 2) and fracture mechanics. This can be expressed as

$$w_c(t) = \frac{4\pi d_s(t)}{(1-\nu_c)(a/b)^{\sqrt{\alpha}} + (1+\nu_c)(b/a)^{\sqrt{\alpha}}} - \frac{2\pi b f_t}{E_{ef}} \quad (10)$$

where  $a$  and  $b$  are the inner and outer radii of the thick-wall cylinder,  $\nu_c$  is Poisson's ratio of concrete,  $E_{ef}$  is the effective elastic modulus of concrete and  $f_t$  is its tensile strength. The key variables in Equation (10) are the thickness of the ring of corrosion products  $d_s$  (Figure 2), which is related to the corrosion rate  $i_{corr}$  and the stiffness reduction factor  $\alpha$ . Details of how to determine these variables are in [20].

With the assumption of Gaussian process, all stochastic parameters of  $w(t)$  required in Equation (8) can be determined. As a numerical example using the values of basic variables in Table 1 and

$w_{cr} = 0.3$  mm, the probability of structural failure due to corrosion induced concrete cracking was obtained and the results are shown in Figure 3. Also shown are the simulation results as verification.

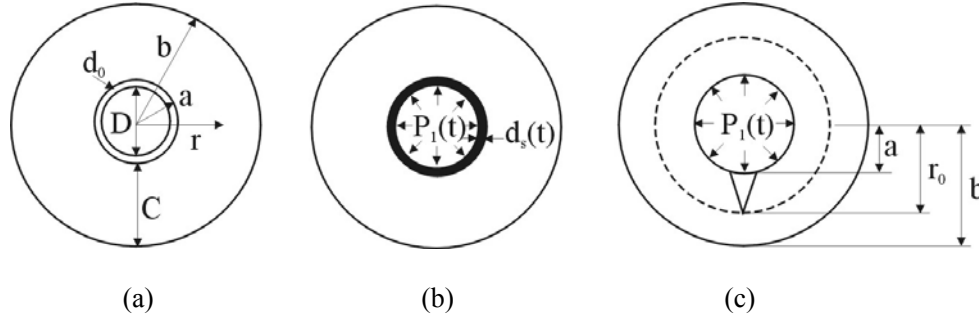


Fig. 2 Schematic of corrosion induced concrete cracking process

### 3.3 Corrosion induced concrete delamination.

The derivation of the probability of structural failure due to corrosion induced concrete delamination is similar to that for cracking failure once a model for delamination is established. Concrete delaminates from the structure when the corrosion induced cracks in the concrete propagate and the neighboring cracks join up to form a fracture plane. How cracks propagate and form a fracture plane depends largely on the geometry and detailing of the concrete section, such as concrete cover, location and diameter of the rebar [21]. When the following geometric condition is met [21], the cracks would propagate between the rebars and form a fracture plane parallel to the surface of concrete cover

$$s - D < 2C \quad (11)$$

where  $s$  is the spacing between rebars,  $D$  is the diameter of the rebar and  $C$  is the depth of concrete cover. Otherwise, an inclined fracture plane would be formed. For the inclined fracture plane, the delamination model in [21] is used in the optimization.

For a parallel fracture plane, the concrete cover separates from the substrate concrete only when a certain crack opening, i.e., crack width, at the fracture plane is reached. When the crack width at any point  $r$  along the crack is larger than a critical limit the layer of concrete (i.e., the cover) delaminates along the fracture plane. Since the corrosion induced crack is tapered from the rebar (see Figure 3(c)) it is considered to be sufficient that when the crack width at the point that two cracks join together, i.e.,  $r = b = s/2$ , is larger than a critical limit the concrete cover delaminates. The limit state function for this condition is

$$G(w_d, w, t) = w_d(t) - w(t) \quad (12)$$

**Table 1** Values of basic variables

Basic variables	Symbol	Mean	COV
Concrete cover	$C$	30 mm	0.2
Diameter of rebar	$D$	10 mm	0.15
Thickness of pore band	$d_0$	12.5 $\mu\text{m}$	-
Effective modulus of concrete	$E_{ef}$	30.12 GPa	0.12
Elastic modulus of steel	$E_s$	200 GPa	-
Compressive strength of concrete	$f_c$	30 MPa	0.15
Tensile strength of concrete	$f_t$	3.0 MPa	0.2
Yield strength of steel	$f_y$	543 MPa	0.15
Corrosion current density	$i_{corr}$	0.0652t+1.0105	0.2
Rebar spacing	$s$	186 mm	-
Coefficient related to type of rust	$\alpha_{rust}$	0.57	-
Density of rust	$\rho_{rust}$	3600 kg/m <sup>3</sup>	-
Density of steel	$\rho_{st}$	7850 kg/m <sup>3</sup>	-
Poisson's ratio of concrete	$\nu_c$	0.18	-
Relative cost for cracking repair	$C_c(0)$	10%	-
Relative cost for delamination repair	$C_d(0)$	20%	-
Relative cost due to rupture	$C_r(0)$	10	-

**Note:** random variables are assumed of normal distribution.

With this limit state function, the probability of structural failure due to corrosion induced concrete delamination  $p_d$  can be determined in the same way as that for cracking failure and hence will not be repeated here. As a numerical example using the values of basic variables in Table 1 and  $w_d = 0.5$  mm [22],  $p_d$  was obtained and the results are shown in Figure 4.

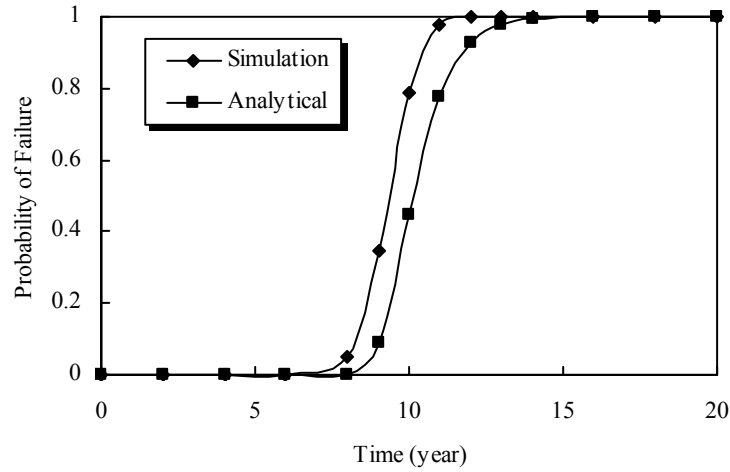


Fig. 3 Probability of structural failure due to concrete cracking

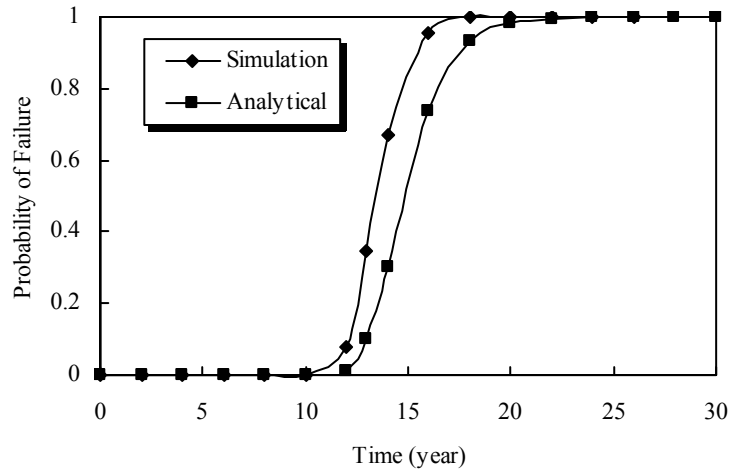


Fig. 4 Probability of structural failure due to concrete delamination

### 3.4 Corrosion induced structural rupture.

Eventually, the rebar corrosion in concrete will lead to the rupture at the critical cross-section of a structural member. The limit state function for structural rupture can be expressed as

$$G(R, R_a, t) = R(t) - R_a(t) \quad (13)$$

where  $R(t)$  is the residual strength at time  $t$  and  $R_a(t)$  is a minimum acceptable strength. Equation (13) represents a downcrossing problem in reliability calculation. Mathematically it has been proved [16] that the formulation and solution of a downcrossing problem are exactly the same as those of upcrossing, when the problem is looked at “upside-down”. Therefore,  $p_r(t)$  can be determined from Equations (5) and (6) with  $R$  replacing  $S$  and  $R_a$  replacing  $L$ .

In analogy to the model for crack width, the residual strength  $R(t)$  can be modeled as



$$R(t) = R_s(t) \cdot \xi_R \tag{14}$$

where  $R_s(t)$  is the residual strength at the critical cross-section of a structural member,  $\xi_R$  is a random variable with  $E(\xi_R) = 1$  and  $\lambda_R$ . The residual sectional strength  $R_s(t)$  can be expressed, in terms of the net area of rebar,  $A_{net}(t)$ , as

$$R_s(\mathbf{E}, t) = f[A_{net}(\mathbf{E}, t)] \tag{15}$$

where  $f[ ]$  is provided by standard concrete design codes (e.g., [23,24]) and  $\mathbf{E}$  is a vector of factors affecting cross-sectional area reduction of the rebar. The most significant factor is the corrosion rate  $i_{corr}$ . The statistics of  $R(t)$  can be determined in the same way as those of crack width and hence will not be repeated.

In Equation (13),  $R_a$ , the acceptable limit for the strength deterioration, is very difficult to decide since the safety is of paramount importance. It is not just a technical issue and there is not much practical experience in this area either. [25] observed that a damage level of 25% in terms of the cross-sectional area reduction of rebar bars seems to be prominent in the real world of corrosion affected RC structures. [26] predicted the service life of corrosion affected RC structures using a more simplistic 30% of rebar area reduction as the failure criterion. In lieu of prescribed acceptable limit,  $R_a$  is taken to be 70% of the original strength in this paper, i.e.,  $R_a = 0.7R_s(0)$ . With the models for both  $R(t)$  and  $R_a$ , the probability of structural failure due to the rupture at the critical section can be obtained and the results are shown in Figure 5.

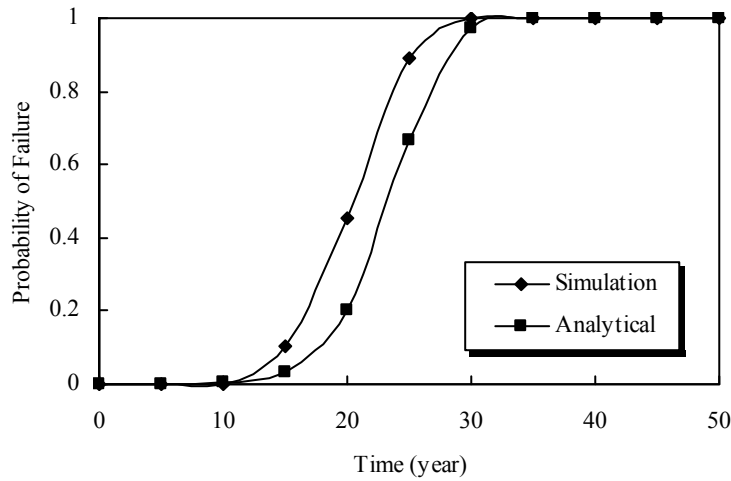


Fig. 5 Probability of structural failure due to sectional rupture

#### 4. Computational Program

Although each term in Equation (1) has been determined individually, the optimization itself is very computationally involved and complex. In this paper a numerical algorithm is developed for the optimization and programmed in MatLab to execute all the computations. The notation used in the algorithm is summarized in Table 2. The computational procedure of optimization is as follows.

**Part I – concrete cracking**

1. At a given time sequence  $t_k$ , select a rebar  $b_l$ .
2. If  $t_k = t_0$  go to next step. Otherwise check whether or not  $b_l$  is repaired at previous time  $t_{k-1}$ . If yes, set  $t_k = t_0$  otherwise  $t_k = t_{k-1} + \Delta t$ .
3. Determine  $\lambda_w$  using Monte Carlo simulation with a reasonably large number of samples (5000 in the example) and the model for crack width  $w(t_k)$  of Equation (10).
4. Determine all probabilistic parameters for crack width  $w(t_k)$ .
5. Calculate  $p_c(t_k)$  for  $b_l$  using Equation (8).
6. Repeat steps 2 to 5 for next rebar, i.e.,  $b_{l+1}$ .
7. Find the maximum of probability of cracking from all rebars, using  $(p_c^{b_l}(t_k))_{\max} = \max\{p_c^{b_l}(t_k), l = 1, \dots, n\}$ .
8. Assign rebar with the maximum probability of cracking  $b_l^{\max}$ .
9. If  $(p_c^{b_l}(t_k))_{\max} \geq p_{c,a}$  repair is needed for rebar  $b_l^{\max}$ . Then assign  $t_c^i = t_k$ ,  $p_c(t_c^i) = (p_c^{b_l}(t_k))_{\max}$ ,  $b_l^r = b_l^{\max}$  and  $r_{b_l^{\max}} = 1$ .

So the rebar that causes cracking at this time is determined and the maintenance follows. With the increase of time, the next rebar causing cracking will be determined and the process continues as follows:

10. Count the total number of repairs for cracking within the maximum time  $t_{\max}$  and denote it as  $n_{mc}^{\max}$  and the corresponding cracking time is  $t_c^i, i = 1, \dots, n_{mc}^{\max}$ .
  11. Determine repair costs of cracking using  $C_{Mc} = \sum_{i=1}^{n_{mc}^{\max}} C_c(t_c^i) \cdot p_c(t_c^i)$ .
  12. Substitute  $C_{Mc}$  in the objective function of Equation (1).
- A subroutine can be programmed to implement the about computations.

**Part II – concrete delamination**

1. First, check geometric conditions with Equation (16) to determine the type of delamination.
2. If it is inclined fracture plane, the model in [21] is used for delamination. Otherwise, model of Equations (9), (10) and (12) is used.
3. Follows the same steps for cracking failure to determine  $p_d$  but replace the variables that indicate cracking  $c$  with those of delamination  $d$ , e.g., replace  $p_c^{b_l}(t_k)$  with  $p_d^{b_l}(t_k)$ .
4. Count the total number of repairs for delamination within the maximum time  $t_{\max}$  and denote it as  $n_{md}^{\max}$  and the corresponding cracking time is  $t_d^i, i = 1, \dots, n_{md}^{\max}$ .
5. Determine maintenance costs of delamination using  $C_{Md} = \sum_{i=1}^{n_{md}^{\max}} C_d(t_d^i) \cdot p_d(t_d^i)$ .
6. Substitute  $C_{Md}$  in the objective function of Equation (1).

**Table 2** Notation used in the algorithm

Notation	Description
$b_l$	Rebar identifier with $l = 1, \dots, n$ ; where $n$ is total number of rebars
$b_l^r$	Rebar $l$ that is repaired
$b_l^{\max}$	Rebar $l$ with maximum of probability of cracking
$n_{mc}$	Number of maintenance due to cracking
$n_{mc}^{\max}$	Maximum number of maintenance due to cracking
$n_{mc}^{opt}$	Optimal number of maintenance due to cracking
$n_{md}$	Number of maintenance due to delamination
$n_{md}^{\max}$	Maximum number of maintenance due to delamination
$n_{md}^{opt}$	Optimal number of maintenance due to delamination
$p_{c,a}$	Acceptable probability of cracking
$p_{d,a}$	Acceptable probability of delamination
$p_{r,a}$	Acceptable probability of rupture
$p_c^{b_l}(t_k)$	Probability of cracking at rebar location $l$ at time $t_k$
$[p_c^{b_l}(t_k)]_{\max}$	Maximum probability of cracking at rebar $l$ and time $t_k$
$r_{b_l^{\max}}$	Indicator which shows rebar $l$ with maximum probability of cracking
$t_c^i$	Time of maintenance due to cracking
$t_c^{i,opt}$	Optimal time of maintenance due to cracking
$t_d^i$	Time of maintenance due to delamination
$t_d^{i,opt}$	Optimal time of maintenance due to delamination
$t_L$	Service life time
$t_k$	Time sequence with $k = 0, \dots, \max$
$t_{\max}$	Maximum time used in optimization
$\Delta t$	Time increment

**Part III – structural rupture**

1. At a given time  $t$ , determine  $\lambda_R$  using Monte Carlo simulation with a reasonably large number of samples (5000 in the example) and the model for residual strength of Equation (15).
2. Determine all probabilistic parameters for residual strength  $R(t)$ .
3. Calculate  $p_r(t)$  with limit state function of Equation (13).
4. Check whether or not  $t = t_{\max}$ . If yes go to next step. Otherwise increase time  $t = t + \Delta t$  and repeat from step 1.
5. For a given  $p_{r,a}$  the service life time  $t_L$  can be determined by  $p_r(t_L) \leq p_{r,a}$ .
6. Determine the cost of structural rupture using  $C_F = C_r(t_L) \cdot p_r(t_L)$ .
7. Substitute  $C_F$  in the objective function of Equation (1).

**Overall - minimizing cost**

1. Calculate the total costs using Equation (1) for each combination of different numbers of cracking ( $n_{mc}^{\max}$ ) and delamination ( $n_{md}^{\max}$ ).
2. Determine the minimum cost from all calculated costs.
3. The corresponding number of crackings and delaminations are the optimal number of repairs for cracking  $n_{mc}^{opt}$  and delamination  $n_{md}^{opt}$  respectively.
4. Determine the corresponding cracking times  $t_c^{i,opt}$  and delamination times  $t_d^{i,opt}$ .

A detailed flowchart for overall computational procedure of minimizing cost is shown in Figure 6 where each part is a subroutine. With this algorithm, a user-friendly window-based program is developed to perform all the computations and displays the results graphically as to be shown in the example.

**5. Numerical Example**

To illustrate the application of the proposed methodology for asset management strategy and in particular the developed algorithm to practical structures, a RC seawall is used as an example. Assume that a segment of the (long) wall is selected for investigation. As can be seen from the models of structural response in each phase of service, a key parameter in the models is the corrosion rate  $i_{corr}$ . In fact, it is an essential parameter for the assessment of corrosion induced structural deterioration. The measurement of it, however, is site or structure specific and its accuracy affects the assessment to a great deal. In this example,  $i_{corr}$  is taken from the measurement on a large scale RC seawall that has been undertaken at the University of Dundee, UK. This is the primary reason to use a RC seawall as an example here. The wall has a dimension of 1000 (wide) x 2000 (high) x 150 (thick). It is subjected to simulated saltwater spray under simultaneous service load of 60% nominal strength. Other variables of the geometry and material properties of the wall are also shown in Table 1. With these inputs, the optimized time and number of cracking and delamination maintenances and the service life can be determined using the developed computer program. The results are shown in Figure 7 (window image). As can be seen, repairs for cracking for different rebar locations and at different times are clearly marked. Same is for delamination. The relative location shown in the figure means it is relative to this segment of the wall.

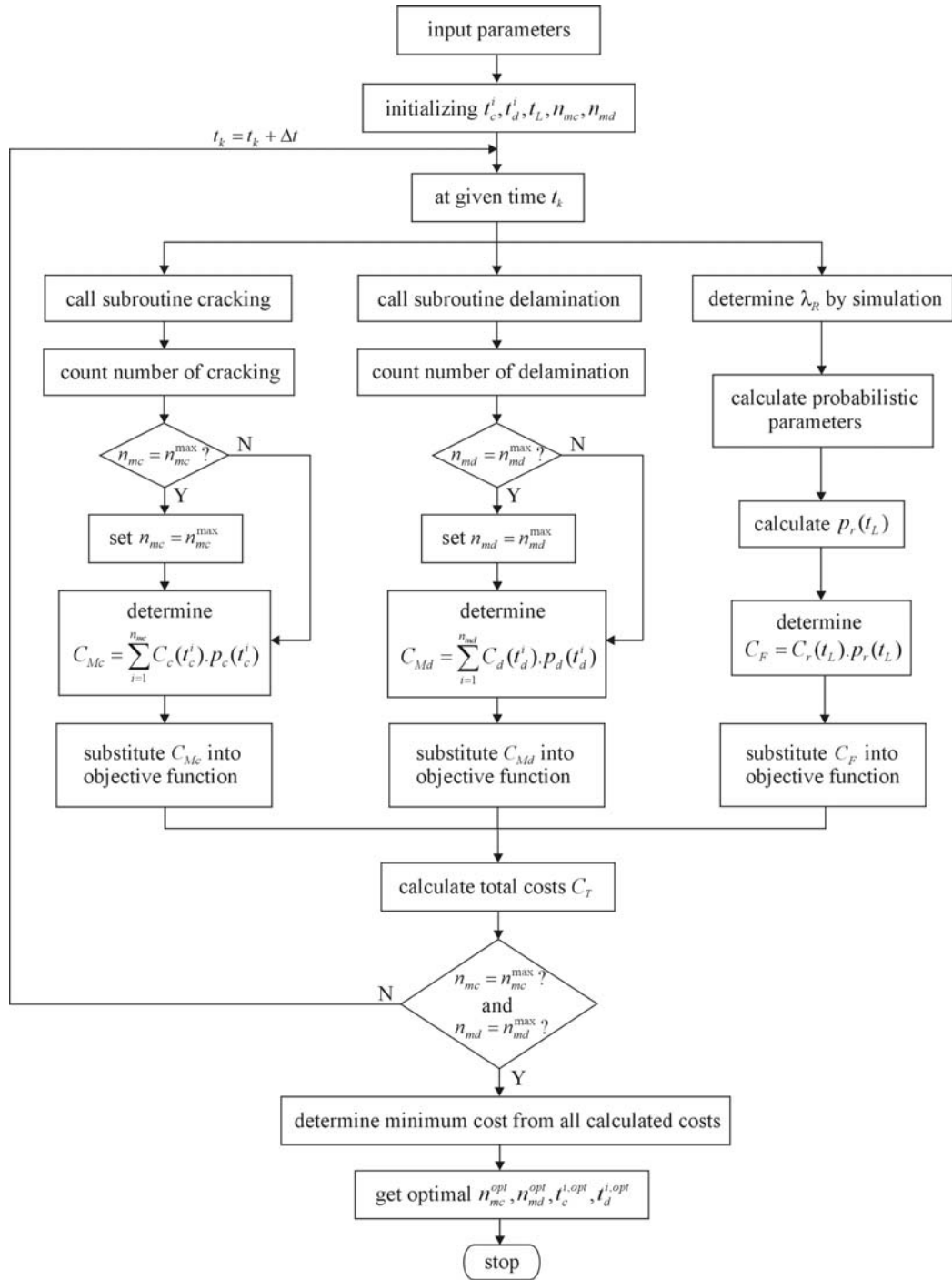


Fig. 6 Overall computational procedure for minimizing total cost

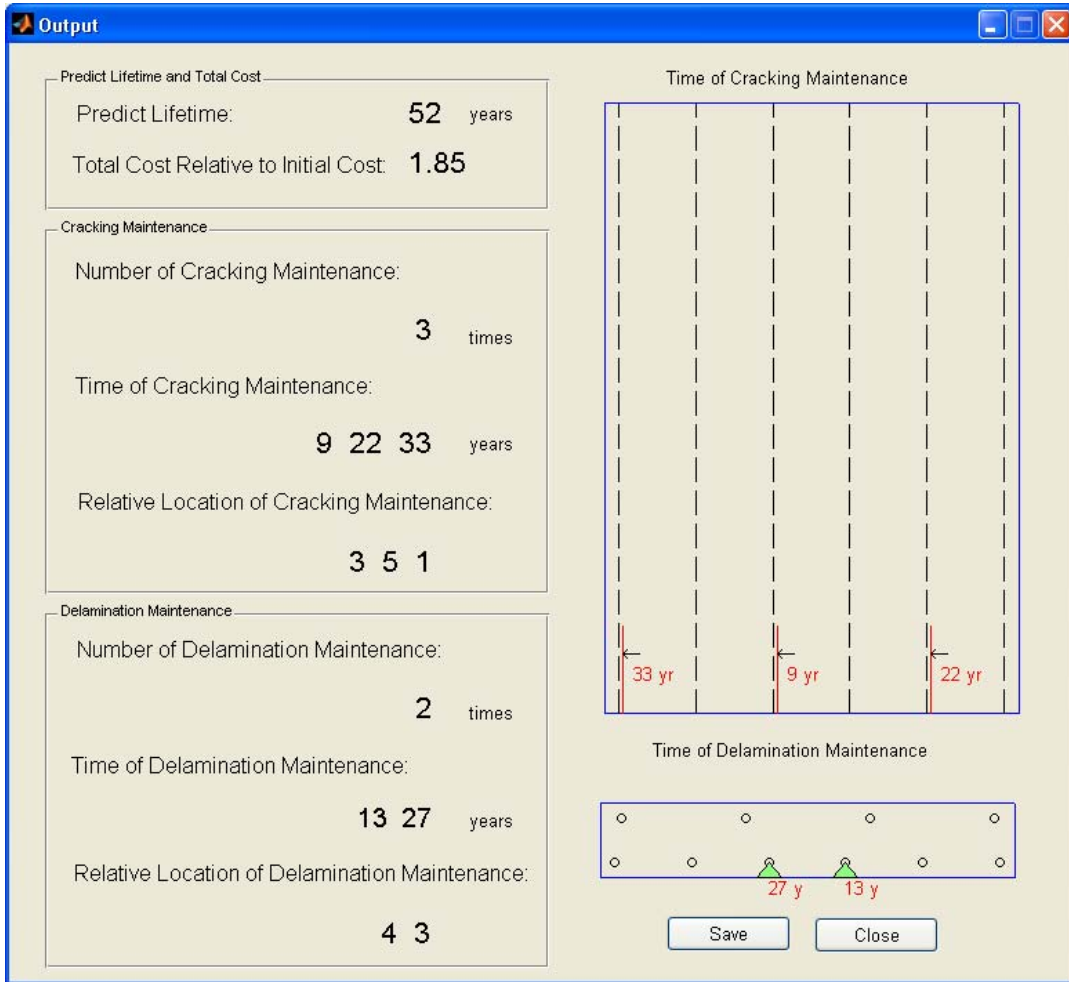


Fig. 7 Outputs of optimization with optimal time and location of cracking and delamination (window image)

The results in Figure 8 demonstrate that the total cost is a function of the number of crackings and delaminations and there exists a minimum total cost for a given accepted risk. This can be the vindication of the formulation of Equation (1). From Figure 9, it is of interest to note that the cost of structural rupture affects the optimal number of cracking and delamination maintenances. Higher structural rupture cost leads to higher minimum total cost when the number of cracking and delamination maintenances is small. However, when the number of maintenances is large, the effect of structural rupture cost on the minimum total cost diminishes as shown in Figure 9. Clearly the information in Figure 7 can well equip asset managers and operators with a rational and practical asset management strategy for corrosion affected infrastructure and thereby achieve the cost-effectiveness in its management.

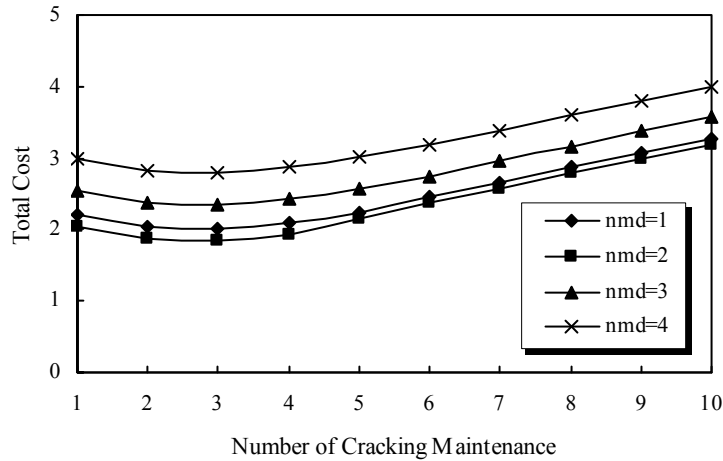


Fig. 8 Effect of number of maintenances on total cost

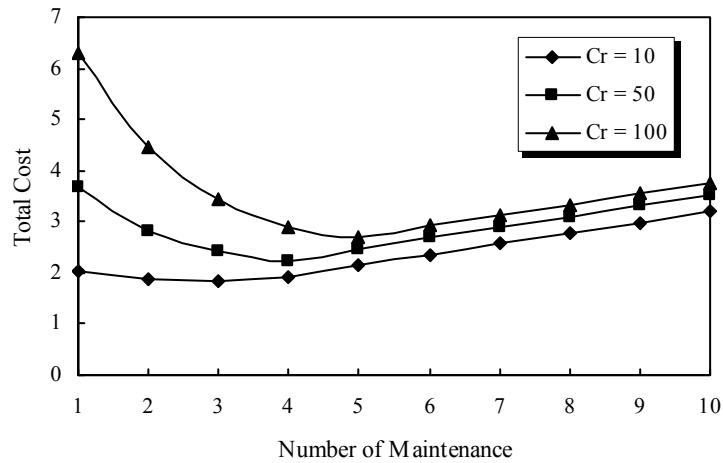


Fig. 9 Effect of cost of structural rupture on total cost ( $n_{md} = 2$ )

## 6. Conclusion

An asset management strategy for corrosion affected concrete infrastructure has been formulated in the paper based on the concept of risk cost optimization. Performance-based models for service life prediction have also been proposed. Using time-dependent reliability methods the risk of attaining the limit state of each phase of service life can be obtained. To facilitate practical application of the formulated strategy, an algorithm has been developed and programmed in a user-friendly manner and on window base. The proposed asset management strategy has the advantage that models used in risk assessment for each phase of service life are related to design criteria used by practitioners and that multiple limit states for structural performance have been considered in the risk cost optimization. It has been found in the paper that there exist an optimal number of maintenances for cracking and delamination that returns the minimum total cost for the structure in its whole life. It has also been found that the cost of structural rupture affects the optimal number of cracking and delamination maintenances. It can be concluded that the proposed asset

management strategy can help structural engineers, operators and managers make decisions with regard to repairs, strengthening and/or rehabilitation of corrosion affected concrete infrastructure.

### Acknowledgements

Financial support from the Royal Academy of Engineering (GRA 10177/93) and EPSRC (EP/E00444X), both of the UK, is gratefully acknowledged.

### References

1. ACI Committee 222, (1985), "Corrosion of Metals in Concrete", *ACI Journal*, **82**, (1), 3 – 32.
2. Broomfield, J, (1997), *Corrosion of Steel in Concrete, Understanding, Investigating & Repair*, E & FN Spon, London.
3. Ben-Akiva, M, Humplick, F, and Madanat, S, (1993), "Infrastructure Management under Uncertainty: Latent Performance Approach", *J Trans. Engrg. ASCE*, **119**, (1), 43 – 58.
4. Engelund, S, Sorensen, J D, and Sorensen, B, (1999), "Evaluation of Repair and Maintenance Strategies for Concrete Coastal Bridges on a Probabilistic Basis", *ACI Materials J.*, **96**, (2), 160 – 166.
5. Enright, M P and Frangopol, D M, (1999), "Maintenance Planning for Deteriorating Concrete Bridges", *Journal of Structural Engineering, ASCE*, **125**, (12), 1407 – 1414.
6. Mori, Y, and Ellingwood, B R, (1993) "Reliability-Based Service-Life Assessment of Aging Concrete Structures", *Journal of Structural Engineering, ASCE*, **119**, (5), 1600 – 1621.
7. Mori, Y, and Ellingwood, B R, (1994), "Maintenance Reliability of Concrete Structures I: Role of Inspection/Repair", *Journal of Structural Engineering, ASCE*, **120**, (8), 824 – 845.
8. Sommer, A M, Nowak, A S, and Thoft-Christensen, P, (1993), "Probability-Based Bridge Inspection Strategy", *Journal of Structural Engineering, ASCE*, **119**, (12), 3520 – 3536.
9. Weyers, R E, (1998), "Service Life Model for Concrete Structures in Chloride Laden Environments", *ACI Material Journal*, **95**, (4), 445 – 453.
10. Stewart, M.G, (2000), *Risk-Based Optimisation of Repair Strategies for Concrete Bridge Decks Considering Cracking and Spalling Limit States and Life Cycle Cost Analysis*, Research Report No. 190.03.2000, Department of Civil, Surveying and Environmental Engineering, The University of Newcastle, Australia.
11. Francois, R, and Castel, A, (2001), "Discussion on Influences of Bending Crack and Water-Cement Ratio on Chloride-Induced Corrosion of Main Reinforcing Bars and Stirrups", *ACI Materials Journal*, **98**, (3), 276 – 278.
12. Li, C Q, (2001), "Initiation of Chloride Induced Reinforcement Corrosion in Concrete Structural Members – Experimentation", *ACI Structural Journal*, **98**, (4), 501 – 510.
13. Li, C Q, (2002), "Initiation of Chloride Induced Reinforcement Corrosion in Concrete Structural Members – Prediction", *ACI Structural Journal*, **99**, (2), 133 – 141.
14. Mohammed and Hamada, (2003), "Discussion on Initiation of Chloride Induced Reinforcement Corrosion in Concrete Structural Members – Prediction by C Q Li", *ACI Structural Journal*, **100**, (1), 133 – 141.
15. Otsuki, N, Miyazato, S, Diola, N B, and Suzuki, H, (2000), "Influences of Bending Crack and Water-Cement Ratio on Chloride-Induced Corrosion of Main Reinforcing Bars and Stirrups", *ACI Materials Journal*, **97**, (4), 454 – 465.
16. Melchers, R E, (1999), *Structural Reliability Analysis and Prediction*, John Wiley and Sons, Chichester.
17. Frangopol, D M, Lin, K-Y, and Estes, A C, (1997), "Life-Cycle Cost Design of Deteriorated Structures", *J. Struct. Engrg., ASCE*, **123**, (10), 1390 – 1401.
18. Li, C Q, and Melchers, R E, (1993), "Outcrossings From Convex Polyhedrons for Nonstationary Gaussian Processes", *J. Engrg. Mech., ASCE*, **119**, (11), 2354 – 2361.
19. Papoulis, A, (1965), *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York.
20. Li, C Q, Melchers, R E and Zheng J J, (2006) "An Analytical Model for Corrosion Induced Crack Width in Reinforced Concrete Structures", *ACI Structural Journal* (accepted).
21. Bažant, Z P, (1979), "Physical Model for Steel Corrosion in Concrete Sea Structures – Application", *Journal of Structural Division, ASCE*, **105**, (ST6), 1155 – 1166.
22. Vu, K A T, and Stewart, M G, (2002), "Spatial Variability of Structural Deterioration and Service Life Prediction of Reinforced Concrete Bridges", *Proc. of Int. Conf. on Bridge Maintenance, Safety and Management - IABMAS 2002*, Barcelona, Spain, July 14-17, CD-ROM.
23. ACI 318, (1999), *Building Code Requirements for Structural Concrete with Commentary*, American Concrete Institute.
24. BS 8110, (1997), *Structural Use of Concrete – Part 1: Code of Practice for Design and Construction*, British Standards Institution.
25. Gonzales, J A, Feliu, S, Rodriguez, P, Lopez, W, Alonso, C, and Andrade, C, (1996), "Some Questions on the Corrosion of Steel in Concrete. Part II: Corrosion Mechanism and Monitoring, Service Life Prediction and Protection Methods", *Materials and Structures*, **29**, 97 – 104.
26. Amey, S L, Johnson, D A, Miltenberger, M A, and Farzam, H, (1998), "Predicting the Service Life of Concrete Marine Structures: An Environmental Methodology", *ACI Material Journal*, **95**, (2), 205 – 214.